VISUAL IMAGERY AND METACOGNITION IN PROBLEM SOLVING

LLOYD DAWE
University of Sydney
JUDY ANDERSON
Bethany College

This paper describes a research project which brings together current interests of the co-investigators. Anderson has been investigating the metacognitive strategies used by junior secondary students, to solve non-routine problems in mathematics. Dawe has been studying the use of visualisation by teacher trainees, in the comprehension and solution of higher order non-routine problems in mathematics. The project examines the introspections of teachers and students as they solve non-routine problems, and the implications for cooperative classroom learning. The increasing use of computer graphics, and graphics calculators in schools and tertiary institutions, has re-awakened interest in the specific role of visualisation in the problem solving process.We are particularly interested in the interaction of visual imagery with metacognitive strategies in non-routine problem solving. The data and its analysis will provide an indepth qualitative case study, of the role of verbal and non-verbal mental activity in comprehending and solving non-routine problems.

Despite the significant progress made through research and curriculum innovation in Australian schools, there are still many teachers who lack confidence in addressing the needs of their students with respect to problem solving ability and mathematical inquiry generally. They have been used to teacher centred approaches which rely heavily on textbooks. Students are still expected to sit and listen, follow demonstrated procedures, and practice routines and skills for reproduction in examination papers. The revolutionary changes taking place, not only with respect to the teaching of mathematics but in the wider context of school reform, has shaken the confidence of many teachers. Previously tried and trusted methods are threatened as the purposes of schooling and preparation for the world of work are redefined. The problem has been exacerbated by higher retention rates in schools, and demands for all students entering the workforce to be competent in clearly identifed areas.

Research studies over the last decade have pointed to metacognitive factors which have a profound influence on the confidence of both students and teachers to solve problems, including beliefs about mathematics as a discipline and how it should be taught and learned (Schoenfeld,1992). Australian researchers have made a very significant contribution to the field - see for example the review by Putt and Isaacs (1992). This study builds particularly on their work. It addresses the problem solving strategies of two groups of students at the University of Sydney, completing their training as teachers of mathematics in secondary schools. The study is intended to be exploratory, and serve as a pilot project for a larger and more rigorous study in 1994-95 which will involve teachers (stage2) and their pupils (stage3). The research has its origins within the broad framework of how teachers think, plan and conceive of their classroom practice. The notion of the reflective practitioner is a particular hallmark of significant educational research and teaching practice (Smith and Lovat, 1990). Such work has made an important contribution to the development of practical strategies to enable teachers to improve their work. In the light of the fact that the quality of teaching is a major educational issue today, critical reflection upon practice is an excellent mechanism through which quality can be enhanced.

In the field of mathematics education, there has been some research which has attempted to address the critical reflection of mathematics teachers on their work. Jaworski (1991) observed and interviewed six secondary teachers who espoused a constructivist philosophy, in an effort to discover the issues the teachers faced as they attempted to put this philosophy into practice. She concluded (p214) that " the teacher working from a constuctivist philosophy will be a reflective practitioner." Constructivists believe that learning is personal and that knowledge is personally constructed within the social context of the classroom (Cobb, 1986). This view of learning contrasts with the more traditional transmission mode of learning whereby the teacher possesses the knowledge which is presented to the students. Learning is assessed by testing to see if particular objectives have been met which means that the products of learning are valued rather than the process of how children learn (Rowland, 1984). If teachers are to become reflective practitioners, traditional views of teaching and learning will need to be challenged.

This project is designed to investigate the introspections of teachers as they solve non-routine problems themselves, and are encouraged to reflect on the implications for their classroom practice. Problem solving most closely resembles the way that mathematics is used in everyday life, and has been identifed as a key competency expected of all young people in preparation for work (Finn,1991). Further, the National Statement on Mathematics for Australian Schools (Australian Education Council,1990) sets goals for all Australian children which encompass problem solving strategies within the more general area of mathematical inquiry. This has become increasingly important for mathematics teachers as school retention rates continue to rise. There is now an urgent need to provide guidance for teachers in problem solving activities for the much wider ability range of the students in their care. Learning to think mathematically must become a focus of research and investigation if we are to meet the goals of these documents.

Within the very large body of problem solving research which has been carried out in the last decade, many studies clearly indicate that teachers need to be aware of their own thinking processes and the strategies they use in solving problems, before they can successfully help students to solve problems too (Schoenfeld, 1992). Problem solving requires the ability to control and organise thinking. This has been referred to as metacognition. The metacognitive strategies which students use when they solve problems can enhance their problem solving performance (Siemon, 1986). It is therefore of great interest to study the strategies students and their teachers use in solving problems. Hart (1991) for example organised problem solving workshops for middle school teachers aimed at improving problem solving abilities by focusing on metacognitive and constructive processes. Schoenfeld (1987) lists three related but distinct categories of intellectual behaviour which have been the focus of metacognitive research. We have specifically applied them to teachers for the purpose of this study.

1. A teachers knowledge of their own thought processes; how
2. How well a teacher controls or self-regulates their problem solving actions
3. The beliefs and intuitions of the teacher; that is, the ideas about mathematics which a teacher brings to their work and how such ideas shape the way the teacher does mathematics.

The introspections of teachers as they go about solving non-routine problems is an important goal of the study. By coming to reflect critically on their own thought processes, how they can be monitored and controlled, their beliefs about mathematics, and what it means to come to know mathematics, it will be a short step to critical reflection on their classroom practice. In particular the role of visual imagery in comprehending a problem and its subsequent solution will be of great interest.

THEORETICAL CONSIDERATIONS

There are two theories of cognition which particularly lend themselves to the design and the interpretation of the results of the research to be carried out.

Schema theory

Research has found that students who have schemata for meaningful problem types, evidenced by the ability to classify problems on the basis of their semantic structures, are better problem solvers than students who do not have knowledge of problem types (Goldin,1992). Schemata are theorised to be abstract knowledge structures for representing generic concepts stored in memory, although there is no agreement in the literature as to how a schema should be defined. In the field of mathematics education Sweller (1989), for example, defines a schema as a cognitive construct that permits problem solvers to classify problems into particular categories which require particular moves for solution. As a result of many studies which have been driven by this theoretical perspective, there is a widespead view that more explicit instruction in representation skills for comprehending the semantic structure of word problems would be beneficial. Lewis (1989), for example, found that training in the use of diagrams to represent the semantic structure in a word problem improved problem solving performance. Further, the training resulted in transfer of new representation skills to related, but more complex, problem solving tasks.

Schema theory has an honoured place in research in mathematics education. Richard Skemp, for example, has developed his schematic theory of mathematics learning over the last 30 years (for an up to date summary of his work see Skemp, 1992). Many other writers in the field of the psychology of mathematics, have made excellent theoretical contributions to our understanding of mental representations of mathematical ideas, and how mathematical knowledge systems are developed. Goldin (1992) has recently attempted to pull the diverse theoretical contributions together into a unified whole. Mental imagery plays a very important role in these theories, and the literature abounds with studies of "image schemata" (including visualisation) of mathematical concepts. Further, in contrast to reading research, schematic approaches to mathematics learning have often incorporated affective representations into their models.

In discussing the impact of schema theory on research in mathematical problem solving, one can hardly pass the work of Sweller and his colleagues. Putt and Isaacs (1992) summarise the contribution these researchers have made to our understanding of the problem solving process and the implications for classroom practice. According to Sweller (1989) the two keys to skilled problem solving performance are schema acquisition and rule automation, the latter being a rule that can be used without conscious processing. This has sharpened the constructivist debate because, as Putt and Isaacs (1992, p223) point out

> The work of Sweller and his colleagues suggests that, at the secondary level, teachers should be focusing on domain specific strategies which will enhance the development of multiple schemata to meet differing problematic situations of school mathematics. Such an approach to problem solving will be much easier to pass on to teachers of mathematics operating in the conventional role of imparters of knowledge and skills, as it requires only small changes to their current practice. The Polya model on the other hand requires radical changes in the belief systems of both teachers and students, as well as substantial role changes on the part of the teacher.

Dual Coding Theory

Like schema theory, dual coding theory is not specifically a theory of how mathematics is learned, but a theory of cognition. The following summary has been taken mainly from Sadoski, Paivio, and Goetz (1991), but a full description of the evolution of the theory and its empirical record, are extensively documented in Paivio (1986,1991). Dual coding theory holds that cognition consists of two separate mental sub-systems, one specialised for the representation and processing of information concerning nonverbal objects and events, and the other specialised for dealing with language. The former is often referred to as the imagery system because it functions include the generation of mental images (visual, auditory, haptic and affective). The language specialised subsystem is referred to as the verbal system. These systems are separate but interconnected, so that they can function independently, in parallel, or in an integrated manner.

The two systems have different organizational and processing characteristics. Information in the verbal system is organised in a way that favours sequential, syntactic processing. Nonverbal information, especially in the visual mode, is organised more in the form of holistic nested sets, with information available for processing in a synchronous or parallel manner. Interconnections between the systems allow for great variety in cognitive activity. Language can evoke imagery, and imagery can evoke language. Such cross system activation is called referential processing. However the connections are not one to one but one to many. Hence, language could referentially evoke numerous images or none at all. Mental images could evoke much language or none at all.

Within system processing is said to be associative. This refers to spreading activation among verbal representations or among images. Thus words or phrases are associated, through experience or learning, with other words or phrases. Likewise, mental images could evoke other (associated) images. A mental image of a tyre for example, might evoke an image of a car and even a driving experience. Paivio (1991:379) points out that individuals differ in the degree to which they have developed representations corresponding to particular words and objects, interconnections between them, and processing skills involved in making use of the available structures and pathways.

Processing in dual coding theory includes external and internal variables. External contexts, situational constraints, instructions and so on, interact with a person's two existing symbol systems, as determined by prior experience and individual differences. Processing may be conscious or unconscious, and often involves the transformation and recoding of representations. However the theory does not include any separate abstracted structure such as a schema or processing under the guidance of a schema. This, it is claimed, enables the theory to better account for the great detail found in the memory for events, as well as the flexible construction and reconstruction of events in memory.

The application of dual coding theory to problem solving

According to dual coding theory, problem solving performance is mediated by the joint activity of the individual's verbal and imagery systems. The relative contribution of each system will depend on the characteristics of the task and the cognitive abilities and habits of the problem solver. The theory predicts that the more concrete and nonverbal the task, the greater the contribution of the imagery system. The more abstract and verbal the task, the greater the contribution of the verbal system. The demands of the task and individual differences variables will determine the degree to which the various dual coding processes (described above) are brought into play. The imagery system contributes richness of content and flexibility in the processing of that content, so that diverse bits of information can be quickly retrieved, compared, evaluated, transformed and so on. Paivio (1986:201) suggests that these attributes may underlie the intuitive leaps of imagination that often characterise creative thinking. He argues that language and the verbal system on the other hand, provide precise means (conceptual pegs) for retrieving images from memory and guiding the processing of retrieved information. Its capacity for sequential organisation in particular suggests that the verbal system contributes logical order to ideation that would not be possible on the basis of imagery alone.

> That logical potential implies, too, that verbal processes may predominate in the later stages of the task sequences, eg, the so called verification stage of discovery, whereas imagery may predominate at earlier stages, at least in some tasks. Paivio (1986: 202)

Kaufmann's (1980) theory of problem solving has much in common with the dual coding theory approach, although he disagrees with some specific points. He emphasises similar functional distinctions between imagery and linguistic representations. He agrees that visual imagery is crucial during the discovery phase of problem solving. In his general theory, the fuctional usefulness of the two systems is related to the novelty of the task. Thus, he sees linguistic representations as more appropriate when the task is familiar, whereas imagery becomes increasingly appropriate and adaptive as the novelty of the task increases.

In the particular case of informing research into the role of visual imagery in problem solving in mathematics, there is much to be learnt from the above analysis. Individuals differ in their preferred strategies and abilities, so that some rely on imagery to solve certain types of problems, while others use a linguistic strategy. This is in accord with dual processing theory. However, there is good evidence to suggest that, for most people, visualisation may play an important role in comprehending a problem, even if they prefer a linguistic strategy in arriving at the solution. Kaufmann's novelty hypothesis would be interesting to test in the context of a wide range of mathematical tasks.

AIMS OF THE PROJECT

1. To investigate the metacognitive strategies used by two groups of student teachers differing in training and mathematical ability, in solving non-routine mathematical problems
   (i) Diploma of Education students
   (ii) Bachelor of Education students in their final year
2. To study the effect of individual learning style on the metacognitive strategies used.
3. To investigate the specific use of visual imagery in comprehension and analysis in the problem solving

process.

4. To see if there is any evidence to suggest that the students' beliefs about mathematics, and in particular beliefs about problem solving, effect problem solving performance.
5. To see if there is any evidence to suggest that emotional factors effect the metacognitive strategies employed.
6. To generate hypotheses for further research.

THE RESEARCH DESIGN

Methodology

The purpose of the research is to study the thinking processes of two small groups of final year mathematics students as they go about solving non-routine problems. The data and its analysis will then provide an indepth qualitative case study, of the role of verbal and non-verbal mental activity in comprehending and solving such problems. It is intended to be a preliminary pilot study, to inform the planning and design of a study of teachers (and finally school students) as they solve non-routine problems.

The study lends itself to an ethnographic research design, and generally follows the methodology adopted by the co-investigator in her successful study of the metacognitive strategies adopted by her year 8 students in solving non-routine problems (Anderson, 1993). In particular, the protocol developed by Schoenfeld (1985) for use with undergraduate students, will be used to address aims 1 and 3 of the study. For Aim 2 we will use the learning style inventory developed by Kirby et al (1988) and the instrument used by Brumby (1982) - see sample below- while Aim 4 requires a special instrument adapted from Anderson (1993). Aims1-5 will be addressed from observation, interviews and student reporting. The problem solving sessions will be videotaped, and students will be asked to participate in the analysis with the investigators by reviewing the recordings.

It is not possible to fully justify the methods we have decided to use here. After a critical review of the methods other researchers have used, and in particular the results of Anderson's work, we have settled on a combination of methods to gather the data in an effort to overcome weaknesses and to strengthen the findings using triangulation. These include non participant observation of the students (working alone, in pairs and in a group of four), written student recordings, audio and video recordings, student responses from video replays, student interviews, and researcher's field notes. It is hoped that a wide variety of methods will inform the methodological decisions for the next stage of the research with teachers. We have also decided to try out a considerable range of problems which include the analysis of mathematical images generated by computer.

The sample

We plan to include two contrasting groups of students at the University of Sydney in the pilot project: the 1993 intake of Dip Ed(sec) mathematics students, and the final year students in the integrated BEd (sec) mathematics program. The two groups differ markedly on a wide range of variables including mathematical ability, work experience, knowledge of how children learn mathematics and practicum experience. However they are roughly the same age. Having reached the end point of their training they will be appointed as four year trained teachers to schools in 1994. It is likely that the very different training the two groups have received, together with differences in academic ability, will produce some interesting contrasts in the data. It is expected that there will be implications for the inservice needs of particular teachers in the workforce, as the competencies identified by the Finn committee are implemented in schools.

The Dip Ed group comprises 28 students, 16 women and 12 men. The undergraduate group completing the integrated degree comprises 33 students, 18 women and 15 men. It is intended to administer the Visual and Verbal Learning Style Inventory (Kirby, Moore and Schofield,1988) to each group to identify those who prefer visual to verbal modes of processing information. This instrument was designed for adult students and is particularly appropriate for the theoretical basis of Dual Coding theory. Although this will provide helpful initial information, it is inadequate for the selection of the sample. We will also administer the instrument developed by Brumby(1982), who investigated differences in cognitive style for qualitative problem solving in biology. She identified two distinct learning styles which have been described as analytic or serialist, and global, holistic or intuitive. The essential characteristic of the analytic style appears to be that a given problem is immediately broken up into its component parts. These are studied step by step as discrete entities in isolation from each other and their surroundings. The second, holistic, style seeks an overall view of a problem. The various parts are integrated and related and seen in the context of their surroundings. Overall 42% of Brumby's sample maintained an analytic style, 8% used only the holistic style and 50% used a combination of both. Brumby described the latter group as "versatile learners".

We will then select a sample of 4 "visual" and 4 "verbal" and 4 "versatile" students from each group to take part in the study. The distinction between the visual/ verbal styles is seen in terms of the type of processing described in the dual processing model, and by such descriptors as simultaneous versus successive, or holistic versus analytic. The versatile group are more likely to use both, to switch from visual to verbal mode depending on the nature of the problem. The sample does not need to be random and we will choose a balance of men and women, taking into account academic background, to participate in the study.

DISCUSSION

With respect to problem solving, strategies which encourage the development of holistic thought processes, linking them to sequential deductive thinking, are likely to result in increased mathematical performance. Versatile learners able to think in holistic and analytic modes, and use them in harmony, are more likely to be successful, particularly at higher levels of mathematics. Whether visual imagery per se plays an additional role in the actual method of solution seems to be a personal matter. We will be interested in getting further data on this interesting question. What does seem to be important is the role of visualisation in comprehension. In parallel to the comprehension of narrative text, reading a mathematical task with understanding may require an automatic imaging, in which parts of the problem are visualised and brought together to form a whole (gestalt) of information (Dawe, 1993).

Previous research has also shown that students need to be aware of and in control of their thinking, ready to switch to a new line of thought when it becomes clear that the present one is going nowhere. This requires sitting back, asking themselves how they are going, and getting a holistic view. This "internal talk" is characteristic of self monitoring and self regulation. The ability to switch one's viewpoint from a local analytical one to a holistic one, in order to be able to place the details as part of a structured whole, is crucial. Such metacognitive abilty is characteristic of good problem solvers.

What then should teachers be doing about these matters in the classroom? How do teachers enable students to develop their powers of mental imagery? How do teachers identify, and monitor, the metacognitive strategies their students use in problem solving? What about the role of affective variables? If our goal is to encourage the development of all these internal systems of representation, should we pay equal attention to each? These and other issues will be discussed in the presentation.

REFERENCES

Anderson, J. (1993) An investigation of the metacognitive strategies which students use while solving non-routine problems , unpublished MEd (hons) essay, University of Sydney.

Australian Education Council (1990) A National Statement on Mathematics or Australian Schools the Curriculum Corporation, Victoria

Brumby, M. N. (1982) Consistent differences in cognitive styles shown for qalitative biological problem solving. British Journal of Educational Psychology 52, 244-257

Cobb, P. (1986) Contexts, goals, beliefs and learning mathematics. For the Learning of Mathematics, 6(2), 2-9.

Dawe (1993) Visual imagery and communication in the mathematics classroom, in Communicating Mathematics: Perspectives from Research and Classroom Practice , W. Stephens, D.J. Clarke, J. Izzard and A. Waywood (eds), ACER, Melbourne (in press)

Finn, T. B. (Chair,1991) Young People's Participation in Post - Compulsory Education and Training Report of the Australian Education Council Review Committee, Canberra; AGPS

Goldin, G.A. (1992) On developing a unified model for the psychology of mathematical learning and problem solving Proceedings of the Sixteenth Conference of the International group for the Psychology of Mathematics Education vol3, 235-261, University of New Hampshire.

Hart, L.C. (1991) Monitoring change in metacognition. In Furinghetti, F. (ed) Proceedings of the 15th Psychology of Mathematics Education Conference, Italy, Vol 11, 141-148.

Jaworski, B. (1991) Some implications of a consructivist philosophy for the teacher of mathematics. In Furinghetti, F. (ed) Proceedings of the 15th Psychology of Mathematics Education Conference, Italy, Vol 11, 213-220.

Kaufmann, F. (1980) Imagery, language and cognition NewYork, Columbia University press

Kirby, J.R., Moore, P.J. and Schofield N.J. (1988) Verbal and Visual Learning Styles Contemporary Educational Psychology 13, 169-184

Lewis, A.B. (1989) Training students to represent arithmetic word problems Journal of Educational Psychology 81, 4, 521-531

Paivio, A. (1986) Mental Representations: A Dual Coding Approach New York, Oxford University Press

Paivio, A. (1991) Images in the Mind: the evolution of a theory New York, Harvester Wheatsheaf

Putt, I.J. and Isaacs, I. (1992) Research in Mathematical Problem Solving B. Atweh and J. Watson (eds) MERGA, 208-230

Rowland, S. (1984) The Enquiring Classroom. London : Falmer Press.

Sadoski, M., Paivio, A. and Goetz, E. (1991) A critique of schema theory in reading and a dual coding alternative Reading Research Quarterly XXVI,4, 463-484

Schoenfeld, A.H. (1985) Mathematical Problem Solving Orlando: Academic press

Schoenfeld, A.H. (1987) "What's all the fuss about metacognition?" in Cognitive Science and Mathematics Education Hillsdale, N.J., Lawrence Erlbaum Associates, 189-215

Schoenfeld, A.H. (1992) "Learning to think mathematically: Problem solving, metacognition and sense making in mathematics". In Grouws, D.A. (Ed) Handbook of Research on Mathematics Teaching and Learning New York, Macmillan, 334-370

Siemon, D. (1986) "Problem Solving- a challenge for change or a passing fad?" in Swinson, K. (Ed) Mathematics Teaching: Challenge for Change Brisbane, Australian Association of Mathematics Teachers, 31-37

Skemp, R.R. (1992) Bringing theory into the classroom: the school as a learning environment for teachers in B. Southwell, B. Perry and K. Owens (eds) Space- the First and Final Frontier Proceedings of the Fifteenth Annual Conference of the Mathematics Education Research Group of Australasia, University of Western Sydney.

Smith, D. and Lovat, T. J. (1990) Curriculum: Action on Reflection 2nd edition, Social Science Press

Sweller, J. (1989) Cognitive technology: some procedures for facilitating learning and problem solving in mathematics and science. Journal of Experimental Psychology, 81,4, 457-466

Useful references which address the role of technology in developing visualisation in advanced mathematics

Boers, M. and Jones, P. (1992) The graphics calculator in tertiary mathematics in B. Southwell, B. Perry and K. Owens (eds) Space- the First and Final Frontier Proceedings of the Fifteenth Annual Conference of the Mathematics Education Research Group of Australasia, University of Western Sydney.

Ruthven, K. (1990) The influence of graphic calculator use on translation from graphic to symbolic form. Educational Studies in Mathematics 21, 5, 431-450

Tall, D. and Thomas, M. (1991) Encouraging versatile thinking in algebra using the computer. Educational Studies in Mathematics 22, 2 (April) 125-147

West, T.G. (1991) In the Mind's Eye: visual thinkers, gifted people with iearning difficulties, computer images and the ironies of creativity, New York, Prometheus Books

Zimmermann, W. and S. Cunningham (1991) eds, Visualisation in teaching and Learning Mathematics The Mathematical Association of America